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DESIGN CURVES FOR A CIRCULAR POLARIZER

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ABSTRACT

A common method of producing a circularly-polarized wave in a constant cross section waveguide consists of exciting two waves which are linearly polarized and mutually orthogonal. Adjusting the relative phase velocity of the two waves enables one to produce a single wave which is circularly polarized. Occasionally it is desired to produce right-hand circular polarization at one frequency and left-hand circular polarization at another. The design parameters are usually the ratio of cut-off wavelengths of the two waves, the length of the polarizing section and the frequencies of operation. The theory of operation and a set of design curves showing the relationship between the parameters are presented.

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A rather common way to produce a circularly-polarized wave consists of exciting two orthogonal modes, having the same phase and amplitude, in a common waveguide and adjusting their phase velocities so that over a drift region, L, their electrical lengths differ by an odd multiple of 90°. This relationship can be expressed mathematically as follows:

$$\left| \frac{L}{\lambda_{g1}} - \frac{L}{\lambda_{g2}} \right| = \frac{2n + 1}{4} \tag{1}$$

where

$$\lambda_{g1} = \frac{\lambda_1}{\sqrt{1 - \left(\frac{\lambda_1}{\lambda_{c1}}\right)^2}} \tag{2}$$

$$\lambda_{g2} = \frac{\lambda_1}{\sqrt{1 - \left(\frac{\lambda_1}{\lambda_{c2}}\right)^2}} \tag{3}$$

where λ_1 = operating wavelength, λ_{c1} and λ_{c2} are the cut-off wavelengths of the modes being considered, λ_{g1} and λ_{g2} are the guide wavelengths. Since the value $\tau = \lambda_{c1}/\lambda_{c2}$ is at our disposal, an appropriate set of design curves can be plotted giving L/λ_1 as a function of λ_1/λ_{c1} as shown in Fig. 1. The use of these curves is rather straightforward; however, a typical example should be considered for clarity. Assume that $\tau = 1.1$ and $\lambda_1/\lambda_{c1} = 0.8$, then $L = 2\lambda_1$ in order to realize a 90° phase shift (i.e., n = 0) between the two orthogonal modes propagating in the common waveguide.

Still of further interest is the design procedures for determining the parameters of the same waveguide when it is desired to produce a circularly-polarized wave of one sense at one frequency, and of the opposite sense at a different frequency. This is accomplished by making the delay between the orthogonal modes equal to 2n + 1 quarter wavelength at one frequency and 2n + 3 quarter wavelength at the other frequency. This relationship is given mathematically as follows:

$$\frac{L}{\lambda_{g1}} - \frac{L}{\lambda_{g2}} = \frac{2n + 1}{4} \tag{4}$$

$$\frac{L}{\lambda'_{g1}} - \frac{L}{\lambda'_{g2}} = \frac{2n + 3}{4} \tag{5}$$

where the prime indicates the particular parameter at a different operating wavelength, λ_2. Eqs. (4) and (5) must be solved simultaneously; therefore, letting $\tau = \lambda_{c1}/\lambda_{c2}$, $\beta = \lambda_1/\lambda_{c1}$ and $\alpha = \lambda_2/\lambda_1$, substitute into Eqs. (4) and (5) and eliminate L. The result is

$$\alpha\left[\frac{\sqrt{1 - \beta^2} - \sqrt{1 - (\tau\beta)^2}}{\sqrt{1 - (\alpha\beta)^2} - \sqrt{1 - (\tau\alpha\beta)^2}}\right] = \frac{2n + 1}{2n + 3} \tag{6}$$

Eq. (6) gives the relationship between three variables and the index n which will yield simultaneous solutions to Eqs. (4) and (5). We can assign values to one of the variables and obtain a relationship between the others assuming n = 0, 1, 2, etc. Since the design parameter τ is most likely to be at our disposal, we will plot λ_1/λ_{c1} vs. α for various values of τ. These are shown in Fig. 2 for n = 0, and in Fig. 3 for n = 1. The value of L/λ_1, as a function of α for various values of τ, is shown in Fig. 2 for n = 0, where

$$\frac{L}{\lambda_1} = \frac{2n+1}{4} \frac{1}{\sqrt{1-\beta^2} - \sqrt{1-(\tau\beta)^2}} \tag{7}$$

A similar set of values are shown in Fig. 3 for n = 1. The use of these curves can best be explained by an example.

Consider $\alpha = 1.1$ and that for design reasons L/λ_1 must equal 3, and we wish to have a 90° difference in phase of the two modes at one frequency and a 270° difference at the other frequency (i.e., n = 0). Referring to Fig. 2, we see that τ must equal 1.05 in order to obtain the desired results and also β must equal 0.865. The minimum value of L/λ_1 would be equal to 2.5 if τ were chosen equal to 1.07. The dashed line shown (Figs. 2 and 3) indicates the minimum value of L for the corresponding value for α. Referring to Fig. 3 where n = 1, notice that the minimum value of L is somewhat smaller than the values given when n = 0. This reduction in L is accompanied by a decrease in the circularity of polarization obtained over a given operating frequency band. It also results in a larger value of τ and requires operation very near the cut-off frequency for one of the modes. This would undoubtedly result in a larger difference in the input impedance of each mode. Since the analysis assumes equal power in both modes and in-phase excitation, it is not clear that increasing n to obtain a shorter physical length, L, will result in a satisfactory polarizer.

Conclusions

A series of graphs enabling one to obtain the parameters necessary to design a circular-polarization transducer have been presented together with a derivation of the equations used. The design procedure is straightforward and the parameters are applicable to any waveguide system so long as two modes of propagation can occur simultaneously and produce fields which are mutually orthogonal.

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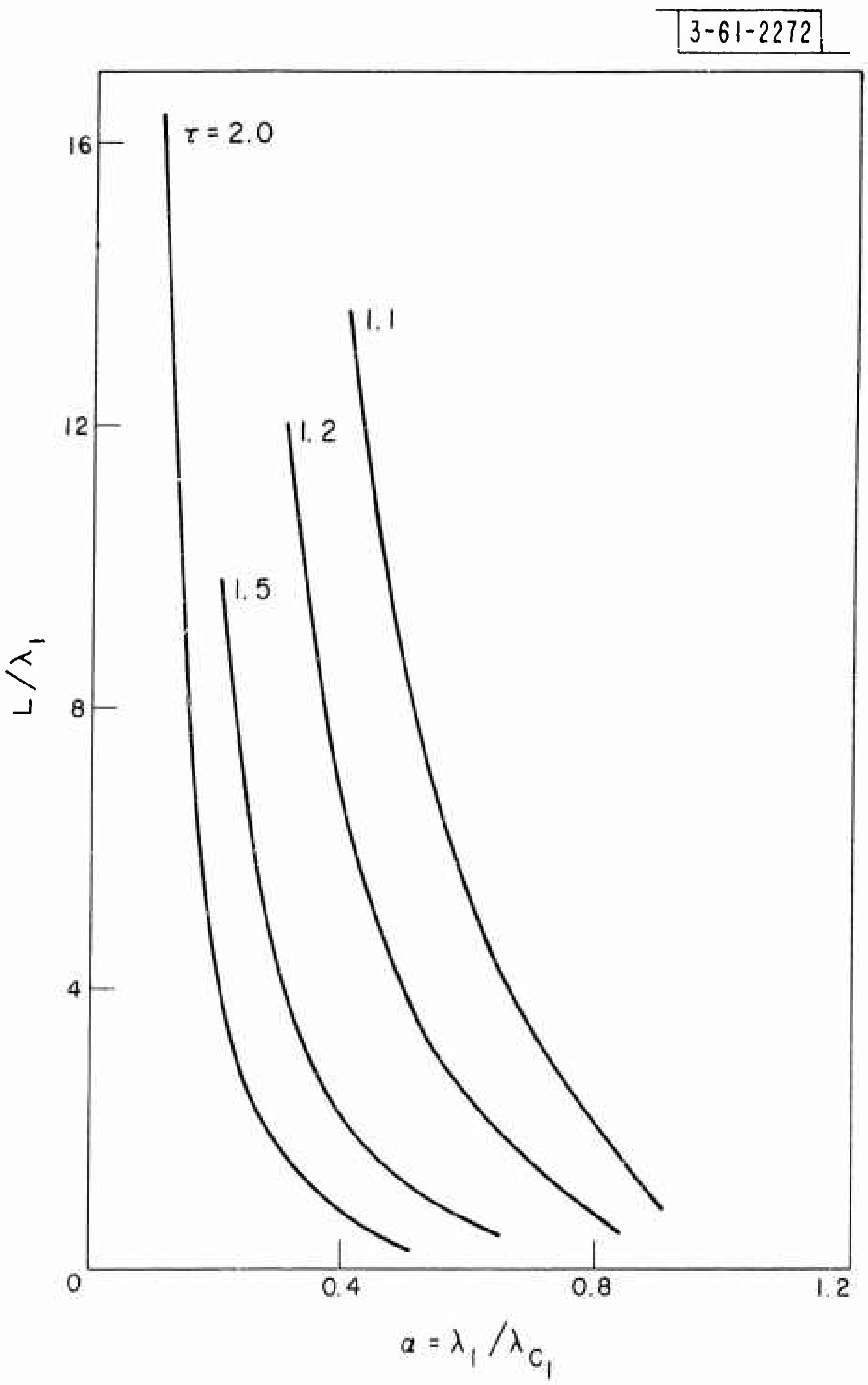
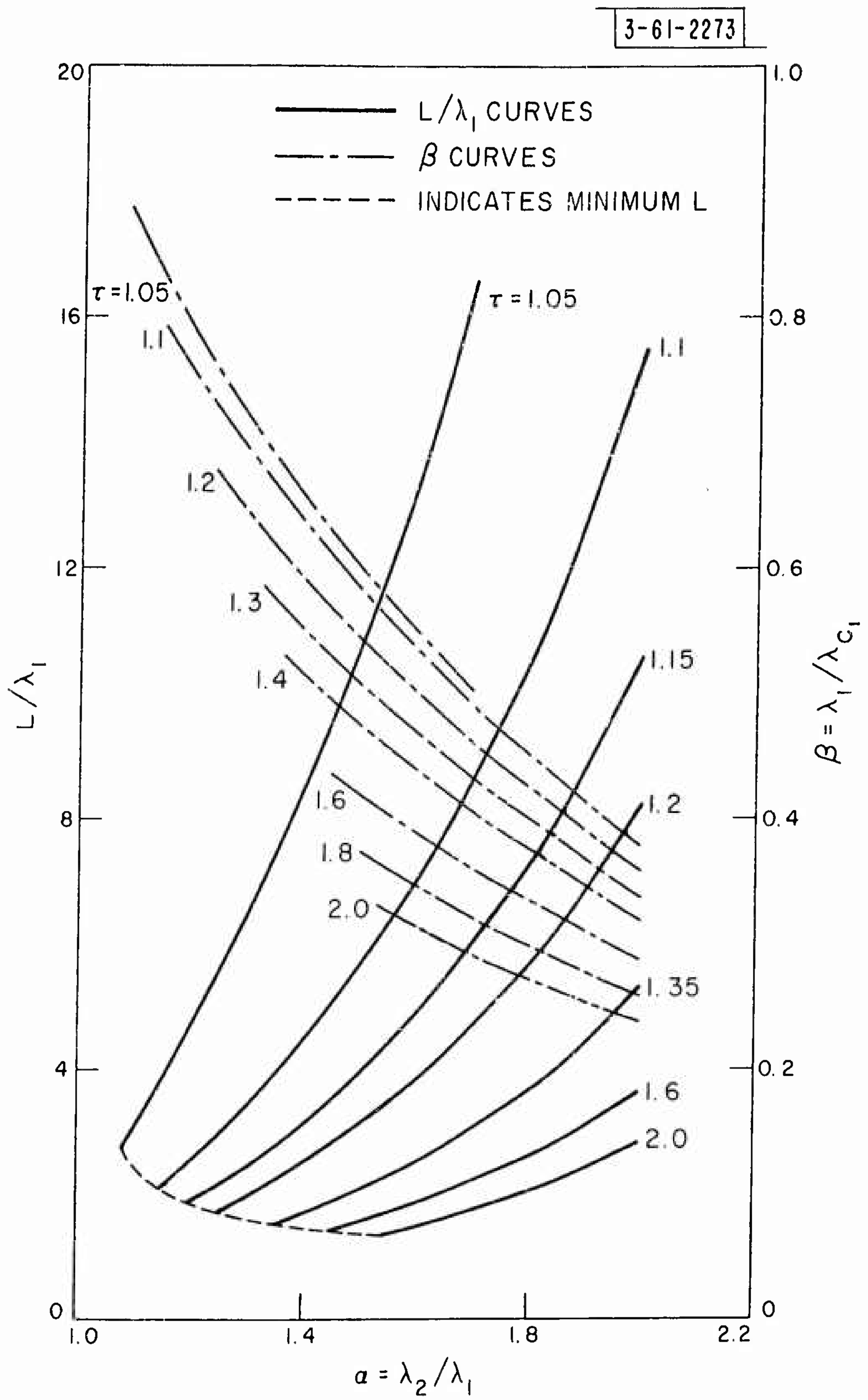


FIGURE 1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L/λ₁ CURVES
β CURVES
INDICATES MINIMUM L
τ = 1.05
1.1
1.2
1.3
1.4
1.6
1.8
2.0
τ = 1.05
1.1
1.15
1.2
1.35
1.6
2.0
20
16
12
8
4
0
1.0
0.8
0.6
0.4
0.2
0
1.0
1.4
1.8
2.2
L/λ₁
β = λ₁/λ_c₁
α = λ₂/λ₁

FIGURE 2

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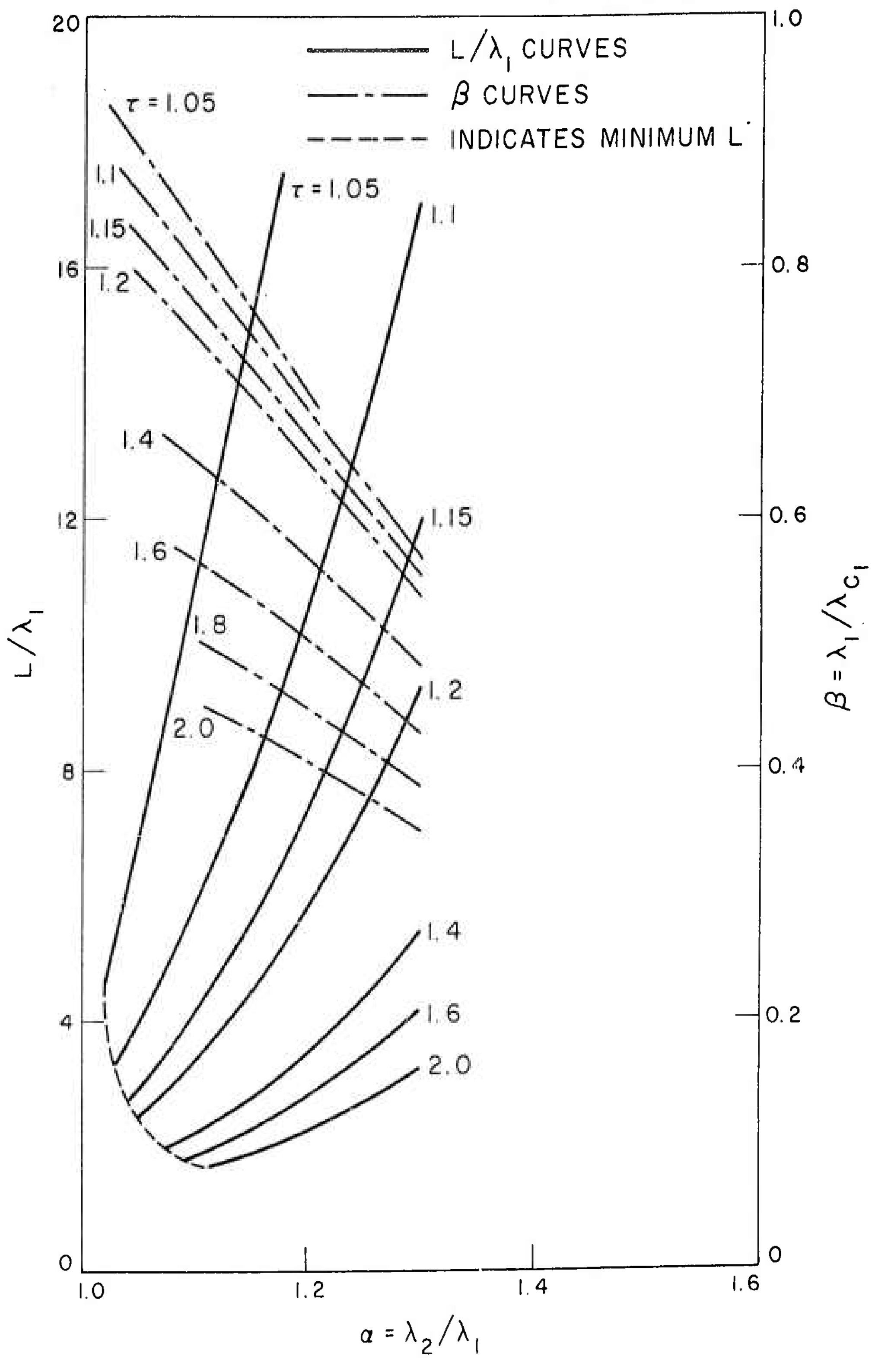

FIGURE 3

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